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Static Voltage Collapse Studies In Power Systems

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Abstract- This paper concentrates on the static voltage stability studies on power systems. The behavior of the system when subjected to gradual and steady increase in system loading is studied. The tool like continuation load flow using the modal analysis technique is used in a systematic way so as to predict the system behavior at different operating conditions. The method devised is a worthy tool to use in power system planning studies from the view point of voltage stability. The main objective is to device suitable computer programs so as to conduct planning studies on a power system from the view point of voltage stability. To do the load flow and identify the weak buses in the system using Eigen value analysis.

Index Terms— Model analysis technique Voltage stability studies, Newton- Raphson Method, Bus and Branch Participation, Eigen value analysis.

I INTRODUCTION

MORE and more voltages related problems are being experienced by modern power systems in India as well as in other foreign countries. It is well known that the voltage problems are closely related to the management of reactive power resources and flows in the system. It has been established that quite a few of the grid failures experienced by different electric utilities have been caused by the voltage collapse phenomenon [10]. The study of a system for voltage instability/collapse conditions will have to be done both from the planning as well as from the operation viewpoints [13]. The present study mainly concentrates on the planning aspects.

Lack of adequate reactive power resources in a power system is a major contributing factor to the process of voltage collapse. As loads in a power system increase, voltage across the network tends to decrease and reactive power, losses increased. The increased reactive power demand would be supplied by voltage regulating devices such as generators or static var compensators, if possible [1]. However, due to physical limitations, such devices cannot supply unlimited amounts of reactive power. Often sustained load growth will result in some source of reactive power, or perhaps a number of such sources, encountering limits, i.e. reaching a physical limitations in the amount of reactive power that they can supply.

Once a reactive power source has reached its maximum limit, it can no longer regulate the voltage. Therefore, sustained load growth results in accelerated voltage decay and hence greater reactive power requirements. This may force other voltage regulating devices to their limits, with subsequent further accelerations in the rate of decline of Voltage. If load is increased further, the point of voltage collapse would soon be reached. This phenomenon has come to be known as voltage collapse.

A voltage collapse process may consist of the following steps: [10]

a) The critical disturbance by increasing transmission line loading would develop and untenable increase of series reactive power losses $I^2 X$.

b) This would cause a sharp voltage reduction at a number of transmission load substations.

c) By reducing loads and provoking over excitation in surrounding units, the voltage reduction would create and initiate stability.

d) But the voltage reduction would also initiate a powerful destabilizing force, automatic on-load transformer tap changing, which would progressively increase the loads, and may even provoke load overshoot.

e) This load increase caused by each tap changing step would progressively reduce the transmission voltage levels.

f) Each voltage reductions would increase series reactive power losses inversely to the voltage cubed.

g) This changing situation could only maintain stability until the first units' over excitations protection functions.

h) Once this occurs, the voltage falls so sharply that system collapse is inevitable.

i) The sustained excess, reactive demand would disrupt inter unit co-ordinations so leading to angle instability as the final cause of breakdown.

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2 MATHEMATICAL MODELLING FOR VOLTAGE STABILITY ANALYSIS

The objective of this paper is to develop the necessary background for the analysis of static voltage collapse problem, establishing the concepts of P-V/Q–V curves (nose curves). Voltage sensitivity to reactive power, number of solutions at different load levels, the point of collapse (PoC) and the condition which occur at voltage collapse in a simple, single line loss-less power system, and then to generalize to a multi-bus system. The issues of sensitivity of voltage to load power on the upper and lower sections of the nose curve and the change of number of solutions as the load power is changed is explored in this paper.

Consider a single line, loss-less system with constant load and fixed sending end voltage, E. This assumption of fixed sending end voltage amounts to limit-less reactive power supply from the generator to keep the generator terminal voltage fixed at E at different load conditions. This section will establish the nose curves, voltage Sensitivity to reactive power, number of solutions at different load levels and the point of collapse in the two-bus system.

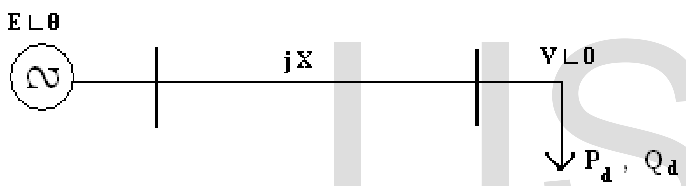

Fig.1. Lossless Two Bus System

The active and reactive power balance equations are:

$$P_d = -P = -\frac{EV}{X} Sin\ \theta \qquad (1)$$

$$Q_d = -Q = -\frac{V^2}{X} + \frac{EV}{X} \ Cos\ \theta \qquad (2)$$

From euqation (1)

$$P_d = \frac{E^2}{2X} \text{Sin } 2\theta \qquad (3)$$

$$P_{d\ max} = \frac{E^2}{2X} \quad \text{at} \quad \theta = 45^0 \qquad (4)$$

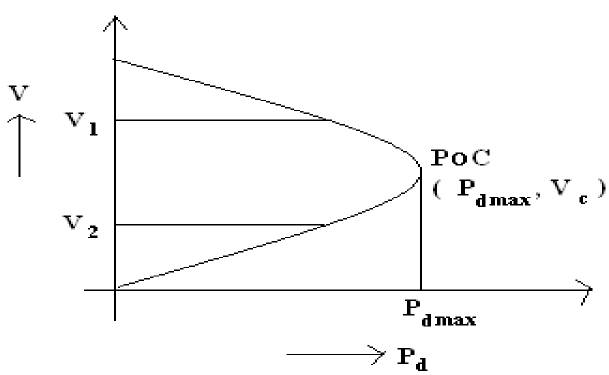

Fig.2. P_d – V Diagram

The Fig. 2 illustrates the following ideas.

a) Sensitivity:

$$\left.\frac{dV}{dP_d}\right|_{V_1} < 0$$

This shows that an increase in P_d at V_1 will cause a decrease in voltage and a decrease in P_d will cause an increase in voltage. This is an expected behavior in normal power system operation and is usually termed as stable solution, upper branch solution or high voltage solution.[1, 2].

$$\left.\frac{dV}{dP_d}\right|_{V_2} > 0$$

The sensitivity of V to P_d at solution point V_2 is opposite to that at V_1. This is called an unstable solution, lower branch solution or low voltage solution.

b) Number of Solutions:

It could be seen that the number of solutions various as P_d is increased. It can be visualized from figure 2 that there will be:

i) 2 Solutions for $P_d < P_d$ max
ii) 1 Solution at $P_d = P_d$ max
iii) No Solution for $P_d > P_d$ max

c) Point of Collapse (Po C):

The point ($P_{d\ max}$, V_C) on the P_d – V diagram, is the Point of Collapse

$$\left.\frac{dV}{dP_d}\right|_{Vc} = \infty$$

This indicates that voltage is infinitely sensitive to Pd at V_c. Similar results can be obtained for Q_d also.

d) Voltage Stability Criteria:

For a single line network, the criterion for voltage stability is stated as follows

$$\frac{dV}{dP} > 0 \qquad \frac{dV}{dQ} > 0 \qquad (5)$$

Where P = - P_d and Q = -Qd.

The point (Q_{dmax}, V_C) is the PoC on Q_d –V diagram. The condition for voltage collapse is thus;

$$\frac{dV}{dQ} = \infty \quad \text{(or)} \quad \frac{dQ}{dV} = 0$$

$$\frac{dV}{dP} = \infty \quad \text{(or)} \quad \frac{dP}{dV} = 0$$

Thus an alternate statement for voltage collapse can be given as: A Power system in steady state is at a point of voltage collapse if some $\frac{\Delta V_j}{\Delta Q_k}$ is unbounded as ΔQ_k tends to zero.

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POWER FLOW EQUATIONS –

Using bus frame of reference, the bus current at i[th] bus is expressed as [3]:

$$I_i = \sum_{k=1}^{n} Y_{ik} V_k \;;\; For\;\; i = 1, 2, ...n \qquad (6)$$

The complex power at i[th] bus

$$S_i = V_i I_i^* = V_i \sum Y_{ik}^* V_k^* \;;\; For\;\; i = 1, 2, ...n \; (7)$$ D

efine

$V_i = |V_i| e^{j\theta}$;

$\theta_{ik} = \theta_i - \theta_k$;

$Y_{ik} = G_{ik} + jB_{ik}$;

$S_i = P_i^{SP} + jQ_i^{SP}$

Then

$$\left.\begin{aligned} P_i(x) &= \sum |V_i||V_k| [G_{ik}\; Cos(\theta_i - \theta_k) + B_{ik} sin(\theta_i - \theta_k)] \\ Q_i(x) &= \sum |V_i||V_k| [G_{ik}\; Sin(\theta_i - \theta_k) - B_{ik} sin(\theta_i - \theta_k)] \end{aligned}\right\} (8)$$

For i=1,2,3, …n.

Define mismatch vector for real and reactive power as:

$P_i(x) = P_i(x) - P_i^{SP}$ (9)

$Q_i(x) = Q_i(x) - Q_i^{SP}$ (10)

Equations (9) and (10) can be expressed in compact form as:

$$f(x) = \begin{bmatrix} p(x) \\ q(x) \end{bmatrix} = 0 \qquad (11)$$

Solution of f(x) = 0 by Newton – Raphson Method

Let $X = X_e + \Delta x$, then by Taylor series expansion.

$F(x) = f(x_e + \Delta x) = f(X_e) + J(X_e)\Delta X$ + higher order terms (12)

Choose Δ x such that f(x) =0 and ignoring the higher order terms of the expansion,

$\Delta x = -J^{-1}(xe)\, f(xe)$ (13)

Where J(xe) the Jacobian evaluated at X_e, Expressed as an iterative scheme, the next estimate of state vector is $X_{i+1} = X_i + \Delta X_I$, the iterative method of solution continues until mismatch functions satisfy a pre-determined tolerance between successive iterations.

3 VOLTAGE COLLAPSE CONDITION IN MULTI-BUS SYSTEMS

The power flow equation in the case of multi-bus system can be written as

$P(\theta, V, \lambda) = P((\theta, V) - P^{SP} - \lambda P^* = 0$ (14)

$Q(\theta, V, \lambda) = Q((\theta, V) - Q^{SP} - \lambda Q^* = 0$ (15)

Where λ is the loading parameter and

$$P^* = -\frac{dp}{d\lambda} \text{ and } Q^* = -\frac{dp}{d\lambda}$$

Linearization of equations (14) and (15) yields,

$$\begin{bmatrix} dp \\ dq \end{bmatrix} = J \begin{bmatrix} d\theta \\ dv \end{bmatrix} + J_\lambda \; d\lambda = 0 \qquad (16)$$

Where J is given by

$$J_\lambda = \begin{bmatrix} \frac{dp}{d\lambda} \\ \frac{dq}{d\lambda} \end{bmatrix} = \begin{bmatrix} -P^* \\ -Q^* \end{bmatrix} = \begin{bmatrix} J_{P\lambda} \\ J_{Q\lambda} \end{bmatrix} \qquad (17)$$

Therefore it gives the direction of loading from equation (16)

$$\begin{bmatrix} d\theta \\ dv \end{bmatrix} = -J^{-1} [J_\lambda] d\lambda \qquad (18)$$

So λ can be increased until J becomes singular. The point of voltage collapse therefore corresponds to the singularity of J, i.e.,det J = 0

VOLTAGE SENSITIVITY

The linearised power flow equation is re-written in a modified form as:

$$\begin{bmatrix} \Delta p \\ \Delta q \end{bmatrix} = \begin{bmatrix} J_{P\theta} & J_{PV} \\ J_{q\theta} & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta|V| \end{bmatrix} \qquad (19)$$

Using equation (16) and assuming that there is no angular instability, i.e., det $J_{P\theta} \neq 0$

If λ is set equal to E, the generator terminal voltage

$dV = - J_{QS}^{-1} (J_{Qe} - J_{Q\theta}\; J_{P\theta}^{-1} J_{Pe})\, dE$ (20)

$$\text{Where } J_{Qe} = \frac{\partial q}{\partial E} \text{ and } J_{Pe} = \frac{\partial P}{\partial E}$$

If λ is set equal to b, the susceptance of SVC at the load bus ($Q_d = -b\, V^2$) and

$dV = J_{QS}^{-1} (J_{Qb} - J_{Q\theta}\; J_{P\theta}^{-1} J_{Pb})\, db = - J_{QS}^{-1} J_{Qb}\, db$ (21)

$$\text{As } J_{pb} = \frac{\partial P}{\partial b} = 0 \text{ and } G_b = \frac{\partial q}{\partial b}$$

Thus, in single line case, in addition to the condition given by equation (5) for voltage stability, the following are also to be satisfied.

$$\frac{dV}{dE} > 0, \quad \frac{dV}{db} > 0 \qquad (22)$$

In a multi – bus system the sufficient condition for voltage stability is that all elements of J_{Qs}^{-1} , $-(J_{Qe} - JQ_\theta\, J_{P\theta}^{-1}\, J_{Pe})$ and $-(J_{Qb} - J_{Q\theta}\; J_{P\theta}^{-1}\; J_{Pb})$ are positive when there is no angular stability problem.

Angle Sensitivity to Change in Parameter

The relationship between voltage sensitivity to parameter λ, A similar relationship can be established between θ and λ. Using equation (15):

$$\begin{bmatrix} J_{PO} & J_{PV} \\ J_{Q0} & J_{QV} \end{bmatrix} \begin{bmatrix} d\theta \\ dv \end{bmatrix} + \begin{bmatrix} J_{P\lambda} \\ J_{q\lambda} \end{bmatrix} d\lambda = 0 \quad (23)$$

From equation (23)

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$dV = -J_{QV}^{-1} (J_{Q\theta}\, d\theta + J_{Q\lambda}\, d\lambda)$ (24)

$d\theta = -(J_{PS}^{-1} (J_{P\lambda} - J_{PV} J_{QV}^{-1} J_{Q}\lambda))\, d\lambda$ (25)

If det $J_{QV} \neq 0$, then det J = 0 implies det J_{PS} = 0 and equation (25) shows that angular sensitivity to parameter λ is infinite when det J = 0.

Thus it is seen that in general angle and voltage stability issues are interrelated.

4 MODEL ANALYSIS FOR VOLTAGE STABILITY STUDIES

A Power system is voltage stable, at a given operating condition, if for every bus in the system, bus voltage magnitude increases as reactive power injection at the same bus is increased. A system is voltage unstable if, for at least one bus in the system, bus voltage magnitude decreases as the reactive power injections at the same bus is increased. In other words, a system is voltage stable if V-Q sensitivity is positive for every bus and unstable if V-Q sensitivity is negative for at least one bus.

The linearised steady state system power voltage equations are given by equation (19)

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = \begin{bmatrix}J_{P\theta} & J_{PV}\\ J_{Q\theta} & J_{Qv}\end{bmatrix}\begin{bmatrix}\Delta\theta\\ \Delta V\end{bmatrix} \qquad (26)$$

Where

ΔP = incremental change in bus real power

ΔQ = incremental change in bus reactive power injection.

$\Delta\theta$ = incremental change in bus voltage angle.

ΔV = incremental change in bus voltage magnitude.

System voltage stability is affected by both P & Q. However, at each operating point P is kept constant and voltage stability is evaluated by considering the incremental relationship between Q and V. Although incremental changes in P are neglected in the formulation, the effects of change in system load or power transfer levels are taken into account by studying the incremental relationship between Q and V at different operating conditions.

let $\Delta P = 0$ then,

$\Delta Q = (J_{qv} - J_{q\theta} J^{-1}_{p\theta} J_{pv})\, \Delta V$ (27)

$= J_R\, \Delta V$

And

$\Delta V = J^{-1}_{R}\, \Delta Q$ (28)

J_R is called the reduced Jacobian matrix of the system; J_R is the matrix which directly relates the bus voltage magnitude and bus reactive power injections. Eliminating the real power and angle part from the system steady state equations allows one to concentrate on the study of the reactive demand and supply problem of the system as well as to minimize computational efforts.

PARTICIPATION FACTORS

Bus Participation Factor:-

The relative participation of bus k in mode is given by bus participation factor.

$P_{ki} = \zeta_{ki}\, \eta_{ik}$

Where P_{ki} indicates the contribution of the i^{th} eigen value to the V _ Q sensitivity at bus k. The bigger the value of P_{ki}, the more λ_i contributes in determining V – Q sensitivity at bus k. For all the small eigen values, bus participation factors determine the areas close to voltage instability. Bus participation factors determine the areas associated with each mode. The size of the bus participation in a given mode indicates the effectiveness of remedial actions applied at that bus in stabilizing the mode. The sum of all the bus participation for each mode is equal to unity.

Branch Participations:-

When the change in reactive power injections in $\Delta Q_{mi,}$ the resulting voltage variations is ΔV_{mi} and the model angle variation is,

$\Delta\theta_{mi} = -J^{-1}_{p\theta} J_{pv}\, \Delta V_{mi}$

With ΔV and $\Delta\theta$ known, the linearised reactive loss variation across transmission branch 1j, ΔQ_{1ji}, and the linearised reactive power output variation at generator gk, ΔQ_{gki}, can be calculated,

Let,

$\Delta\theta_{1maxi} = \max_j \Delta Q_{1ji}$,

$\Delta\theta_{gmaxi} = \mathrm{Max}_k\, \Delta Q_{gki}$,

The participation factor of branch i_j to mode I is defined as,

$$P_{1ji} = \frac{\Delta Q_{1ji}}{\Delta Q_{1maxi}} \qquad (29)$$

Branch Participations thus indicate, for each mode, branches consume the most reactive power for a given for a incremental change is reactive load. Branches with high P_{1ji} are those which cause mode I to be weak. The branch participation thus provides valuable information regarding.

i. Remedial actions in terms of transmission branch enhancement and redistributing the power flow to alleviate the loading on that branch,
ii. Criteria for contingency selection,

The participation factor of generator *gk* to mode *i* is defined as, incremental change in system reactive loading i.e

$$P = \frac{\Delta Q_{gki}}{\Delta Q_{gmaxi}} \qquad (30)$$

Generator participations provide important information's regarding proper distribution of reactive reserves among all

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machines in order to maintain an adequate voltage stability margin.

5 TEST SYSTEM AND RESULTS

A Computer program has been developed to conduct the modal analysis of the system. The program is tested with the small system shown in Fig. 3 the test is conducted in such a way that, the operating points are varied by implementing a step change in reactive power at bus 3 (in steps of 0.05 p.u), starting from 0 p.u and incrementing until, the load flow fails to converge.

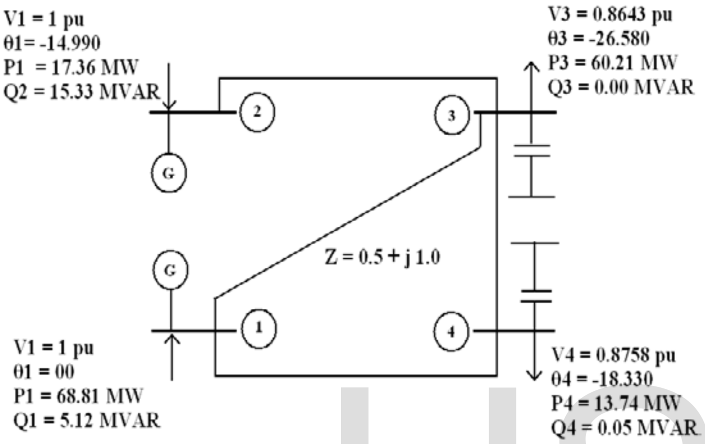

Fig.3. Four Bus Two Generator Test System

The results obtained at different operating conditions are presented in the Table.1.

OPERATING POINT

TABLE I

Model Analysis Results at Different Operating Points

Q_3 = 0.10 pu				Q_3 = 0.15 pu			
Voltage Sensitivity				Voltage Sensitivity			
BUS 3		0.7324		BUS 3		2.94671	
BUS 4		1.065		BUS 4		1.99318	
Eigen Value		Eigen Value		Eigen Value		Eigen Value	
0.7113		2.5472		0.1557		1.1910	
BUS PARTICIPIATION				BUS PARTICIPIATION			
Bus No	Parti.	Bus No	Parti.	Bus No	Parti.	Bus No	Parti.
3	0.3455	3	0.6644	3	2.9467	3	0.4234
4	0.6644	4	0.3355	4	1.9931	4	0.2245
BRANCH PARTICIPIATION				BRANCH PARTICIPIATION			
Br. No.	Br. Parti	Br. No.	Br. Parti	Br. No.	Br. Parti	Br. No.	Br. Parti
1-3	0.5191	1-3	0.8000	1-3	1.0000	1-3	0.005
1-4	1.0000	1-4	0.3928	1-4	0.4855	1-4	0.805
2-3	0.6489	2-3	1.0000	2-3	02403	2-3	-0.7124
3-4	0.6010	3-4	-0.5099	3-4	-0.6430	3-4	1.000

6 CONCLUSION

The fundamental aim behind this paper is to develop some tools which help in predicting the behavior of a complex system with respect to system wide voltage related performance as the loading on encounter their limits. The thesis work done establishing of effect of reactive power on the voltage stability of a power system. The work done is established on a sample 4 bus test system.

The Eigen value analysis method normally known as the modal analysis is used for this study. It is observed that as and when the reactive power is increased on a PQ bus, the minimum value of the Eigen values is decreasing which indicates that the stability of the system is decreasing and when the system is unstable the Eigen value becomes zero.
The study was supported by many other factors such as the bus and branch participation factor and voltage sensitivity which gives a measure of the system stability.

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